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Capillary hydrodynamic effects in high magnetic fields

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A set of hydrodynamic equations has been applied to processes occurring in non-conductive fluids placed into magnetic fields. The equations are valid for equilibrium magnetization within the framework of a continuous medium. The ranges of physical parameters have been evaluated for which magnetization of a fluid should be taken into account in problems concerning the determination of equilibrium forms, and flows and their stability. The conclusion has been drawn that magnetization of natural fluids in these problems must be taken into consideration for fluids in high (exceeding 1 T) magnetic fields. As examples, the solutions of several typical problems concerning the equilibrium surface of capillary fluids and their stability in external magnetic fields have been considered. Hydrodynamic effects related to magnetization of natural capillary fluids in high magnetic fields are studied with due regard for the above solutions. Hydrodynamic effects in para- and diamagnetic fluids have been studied and the common and distinctive features of these effects discussed.

1. Introduction

Strong magnetic fields are known to exist in both nature and technology. The latter uses hydrodynamic processes occurring in constant magnetic fields as strong as 20 T. The trend is to create even stronger fields: pulsed fields can be as strong as 10^3 T. In astrophysics, fields even substantially stronger than 10^3 T are considered (Knoepfel 1970). When applied to a fluid, a magnetic field also causes, in addition to hydrodynamic forces (pressure, viscosity, and surface tension), those of a magnetic origin, the so-called ponderomotive forces, and forces due to magnetization. Magnetohydrodynamics is the study of the effects caused by pondermotive forces. Since there is a trend to use stronger magnetic fields, the study of hydrodynamic phenomena associated with the magnetization of natural fluids in strong magnetic fields (of the order of several T and higher) becomes more and more important.

In hydrodynamics, magnetization gives rise to four effects, namely, the appearance of body force $\mu_0 M\nabla H$ acting on an element of a fluid; a jump in surface pressure $\frac{1}{2}\mu_0\{(\boldsymbol{Mn})^2\}$; the magnetocaloric effect; and the dependence of physical coefficients (viscosity, thermal conductivity, and heat capacity) on magnetization. These factors are very important in the mechanics of fluids – in some ranges of physical parameters these effects start playing a noticeable role.

A body force $\mu_0 M\nabla H$ and a jump in pressure on the surface $\frac{1}{2}\mu_0\{(\boldsymbol{Mn})^2\}$ change the equilibrium shape and flow of fluids: a solitary droplet and the cross-section of a fluid jet elongate in the direction of the lines of force of the magnetic field; a plane surface of the fluid exhibits instability in a transverse magnetic field, etc.

The effect of fluid magnetization on the equilibrium shape of non-electroconductive capillary fluids in external magnetic fields and on the stability conditions of such shapes are studied in the present work, with special attention being paid to the effect of balance of magnetization and capillary forces.

The above typical effects on the mechanics of fluids in strong magnetic fields may be studied using the usual viscous stress tensor for a Newtonian fluid, whose intrinsic magnetic moment $\boldsymbol{M}$ is always parallel to the strength vector H of the magnetic field.

2. Set of basic equations

A wide class of known motions of fluids is described, within the above model of an incompressible fluid with equilibrium magnetization, by the following set of equations (Batchelor 1967; Neuringer & Rosensweig 1964; Berkovsky, Bashtovoi & Vislovich 1985):

$$\left.\begin{aligned} &\rho\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \eta\boldsymbol{\nabla}^2\boldsymbol{v} - \boldsymbol{\nabla}p + \rho\boldsymbol{g} + \mu_0 M\boldsymbol{\nabla}H; \\ &\operatorname{div}\boldsymbol{v} = 0; \quad \operatorname{rot}\boldsymbol{H} = 0; \quad \operatorname{div}\boldsymbol{B} = 0. \end{aligned}\right\} \tag{1}$$

On the free boundaries of a fluid

$$\{p\} = \sigma(K_1 + K_2) + \tfrac{1}{2}\mu_0\{(\boldsymbol{Mn})^2\}; \tag{2}$$

on all the boundaries

$$\{\boldsymbol{H}\}\times\boldsymbol{n} = 0, \quad \{\boldsymbol{B}\}\boldsymbol{n} = 0, \tag{3}$$

where $\{A\} = A - A_e$ is the surface jump of A in the fluid and the surrounding gas (A_e); p, v, M, and ρ are pressure, velocity, magnetization, and the density of the fluid, respectively, B is the magnetic induction, H is the magnetic field strength, μ_0 is the magnetic permeability of a vacuum (magnetic constant), σ is the coefficient of surface tension of a fluid, and K_1 and K_2 are the curvatures of normal sections of the surface.

Transforming the set of equations (1)–(3) to dimensionless form by using characteristic length L, velocity $v = U_0$, magnetic field strength H_0, magnetic field induction $B = B_0 = \mu_0\mu H_0$, time $t = L/U_0$, and pressure $p = \rho g L$, we arrive at

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\boldsymbol{\nabla}^2\boldsymbol{v}}{Re} - \frac{\boldsymbol{\nabla}p}{Fr^2} + \frac{Bo}{We}\boldsymbol{\gamma} + \frac{Bo_{\mathrm{m}}}{We}M\boldsymbol{\nabla}H,$$

$$\operatorname{div}\boldsymbol{v} = 0, \quad \operatorname{rot}\boldsymbol{H} = 0, \quad \operatorname{div}\boldsymbol{B} = 0.$$

The conditions at the free boundaries are

$$Bo\{p\} = (K_1 + K_2) - \tfrac{1}{2}We_{\mathrm{m}}\{(\boldsymbol{Mn})^2\},$$

and those at all the boundaries are

$$\{\boldsymbol{H}\}\times\boldsymbol{n} = 0, \quad \{\boldsymbol{B}\}\boldsymbol{n} = 0.$$

The above set of equations is dependent on several dimensionless criteria, namely, the Froude number $Fr = U_0/(gL)^{\frac{1}{2}}$, the Reynolds number $Re = U_0 L/\nu$, the Bond number $Bo = \rho g L^2/\sigma$, the magnetic Bond number $Bo_{\mathrm{m}} = \mu_0 M_0 G L^2/\sigma$, and the Weber number $We = \rho U_0^2 L/\sigma$. Here G is the characteristic value of the magnetic field strength gradient $\boldsymbol{\nabla}H$, and $\boldsymbol{\gamma} = \boldsymbol{g}/g$. We have introduced the magnetic Weber number $We_{\mathrm{m}} = \mu_0 M_0^2 L/\sigma$ (M_0 is the characteristic value of magnetization M), an analogue of the Weber number equal to the ratio of the dynamic pressure difference ρu_0^2 to the capillary pressure, σ/L. The magnetic Weber number is the ratio of the characteristic difference in surface pressure $\mu_0 M_0^2$, due to a jump in fluid magnetization M, to capillary pressure σ/L.

3. Evaluation of dimensionless criteria

It is seen from the above set of equations that magnetohydrodynamic effects due to fluid magnetization are proportional to the magnetic Bond and Weber numbers, Bo_m and We_m, whence some general recommendations concerning physical problems in which the magnetic properties of fluids should be necessarily taken into account may be deduced.

When studying surface effects, the magnetic properties should be considered in all the cases where the magnetic Bond and Weber numbers are large enough, i.e. are of the order of unity or larger, $Bo_m \gtrsim 1$ or $We_m \gtrsim 1$. In particular, for a large Weber number it is necessary to take into account the jump in pressure due to magnetic forces at the free boundaries (2). The effect of the body force $\mu_0 M \nabla H$ in the gravitational field should be taken into account if this force is comparable with the gravity force, i.e. $Bo_m \gtrsim Bo$.

The evaluation of the Bond and Weber numbers shows that for fluids the surface effects associated with their magnetization become important in strong magnetic fields ($B \sim 8$ T, $|\nabla B| \sim 3$ T/m) created by strong currents ($I \sim 10^7$ A) (Knoepfel 1970). Thus, for example, in a field created by a linear conductor ($I = 10^7$ A) at a sufficient distance from the latter ($a = 1$ m) a natural fluid with 'moderate' magnetic properties (magnetic susceptibility $|\chi| = 10^{-4}$) and considerable surface tension $\sigma = 10^{-1}$ N/m corresponds to magnetic Bond numbers of $Bo_m = \mu_0 \chi I^2/(4\pi^2 \sigma a) \approx 10^4$. In the field $H = 10^7$ A/m a small volume of such a fluid (with the characteristic dimension $L = 10^{-1}$ m) corresponds to the magnetic Weber number $We_m \approx 1$ for which the jump in pressure due to magnetic forces should be taken into account. For larger volumes of the fluid (with characteristic dimension $L = 1$ m), the Weber number increases to $We_m = 10$. For fluids with a lower coefficient of surface tension (which takes place, for example, at high temperatures) $\sigma \sim 10^{-3}$ N/m, the Bond and Weber magnetic numbers take the above-indicated high values in magnetic fields that are weaker by an order of magnitude.

Surface magnetic effects are of great importance for fluids used in heat- and mass-transfer devices in strong magnetic fields. A typical cooling agent in strong magnetic fields is helium (at temperatures of the order of several K (Knoepfel 1970)). Experimental data (Kay & Laby 1966) lead to the assumption that under these conditions liquid helium behaves as a weak diamagnetic substance with magnetic susceptibility $|\chi| \sim 10^{-6}$ and surface tension $\sigma = 0.35 \times 10^{-3}$ N/m. In the calculations of the equilibrium helium surfaces in uniform magnetic fields one need only take into account the magnetization (i.e. $We_m \gtrsim 1$) for strong magnetic fields ($B_0 \approx 10$ T) and large helium volumes ($L \gtrsim 1$ m). In non-uniform magnetic fields, e.g. in the field of a conductor with current I, magnetization should be taken into account (i.e. $Bo_m \gtrsim 1$) over a wide range of the parameters: in the fields of currents nowadays accessible in technology ($I = 10^7$ A) it should be taken into account for almost any helium volume ($L \lesssim 10^2$ m) while for the fields due to 'moderate' currents ($I = 10^6$ A) this should be done for volumes as large as that of a helium bath ($L \lesssim 1$ m). For such volumes ($L \lesssim 1$ m) the magnetic forces are comparable with or exceed the gravitation forces in the fields due to 'moderate' currents, $I = 10^6$ A. In the vicinity of the conductor proper ($L \sim 0.3$ m) carrying a strong current, magnetic forces are essentially stronger than gravitation ones ($Bo_m/Bo \sim 10^3$) and therefore the wetting with helium of a conductor carrying a strong electrical current is determined mainly by the ratio of magnetic to capillary forces. The evaluation of these quantities for other diamagnetic fluids with $|\chi| \sim 10^{-5}$–10^{-6} (liquid hydrogen, neon, argon, water) gives values of the same order of magnitude. For paramagnetic fluids (with higher magnetic susceptibilities, e.g. for liquid oxygen with $\chi \sim 3.4 \times 10^{-3}$ at 70 K), such effects are observed in weaker fields.

In order to illustrate the hydrodynamic behaviour of fluids in strong magnetic fields we shall examine several typical problems on the equilibrium position of a fluid in a magnetic field and its stability.

4. Diamagnetic fluids in non-uniform fields

Experiments (Kaye & Laby 1966) show the wide applicability of the linear dependence, $M = \chi H$, to fields up to $B \sim 10$ T for which the magnetic properties of diamagnetic fluids (DMFs) have been studied. For simplicity, we restrict ourselves to the linear dependence $M = \chi H$, since this assumption will not affect the qualitative conclusions.

A DMF in a non-uniform magnetic field is subjected to the action of a body magnetic force $\mu_0 M \nabla H$ directed towards a weaker field. In the non-uniform field of a linear conductor with a current in a coaxial cylindrical clearance, such a force is directed radially outwards from the conductor. There exists a solution of the set of equations (1)–(3) which corresponds to a droplet of the DMF adjacent to the inner side of the cylindrical clearance (figure 1). The free surface of the droplet, $r = \xi(z)$, has an axial symmetry and therefore does not distort the magnetic field due to the current. This magnetic field has the form $H = [0, H_\phi = I/(2\pi r), 0]$ over the entire space. Using the explicit form of the surface curvature in cylindrical coordinates, we arrive at the following equation for the droplet shape:

$$(1+\xi'^2)^{-\frac{3}{2}}\left(\frac{1+\xi'^2}{\xi} - \xi''\right) = \frac{\mu_0}{\sigma}\int_0^{H(\xi)} M\,\mathrm{d}H + C, \tag{4}$$

where the prime denotes differentiation with respect to z.

Taking into account the symmetry of the problem, we may limit ourselves to the consideration of the upper quarter of the droplet and write the boundary conditions for ξ as

$$\xi(0) = a, \quad \xi'(0) = 0, \quad \xi(z_0) = R, \quad \xi'(z_0) = \tan\gamma. \tag{5}$$

The minimum distance between the conductor centre and the inner surface of the droplet, a, and its half-length z_0 are to be determined. The wetting angle γ is taken to be small ($\gamma \ll 1$). Three unknown constants (C, a, z_0) and two constants obtained by integration of (4) are determined from the four boundary conditions (5) and the condition of the droplet volume constancy

$$V = 2\pi \int_0^{z_0} (R^2 - \xi^2)\,\mathrm{d}z = \text{const.} \tag{6}$$

If the thickness of the droplet is small, i.e. $\delta = (R-a)/R \ll 1$, then, within the terms of the order of δ^2, (4) takes a simple dimensionless form

$$\frac{\partial^2 \epsilon}{\partial y^2} - (Bo_m - 1)\,\epsilon = -C_1, \tag{7}$$

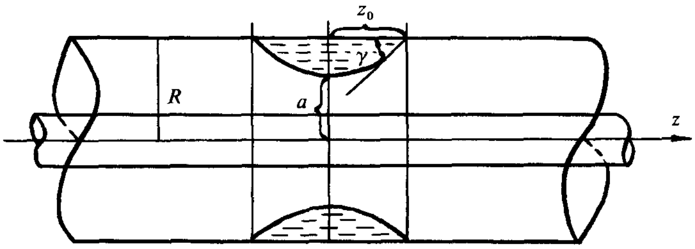

FIGURE 1. A droplet of a diamagnetic fluid in the magnetic field of a cylindrical conductor carrying a current. The droplet is adjacent to the inner surface of a cylindrical clearance coaxial with the conductor.

where the magnetic Bond number, $Bo_m = \mu_0 M_0 G R^2/\sigma$, is determined via the characteristic gradient of the magnetic field strength

$$G = \frac{1}{2\pi R^2}, \quad \epsilon = \frac{a-\xi}{R-a}, \quad y = \frac{z}{R}.$$

The solution of this equation, e.g. for $Bo_m > 1$, has the form

$$\left.\begin{aligned} &\epsilon = \frac{\coth\beta y_0 - \coth\beta y}{\coth\beta y_0 - 1}, \quad \tanh(\beta y_0) = \left(1 + \frac{\beta^2 V}{4\pi R^3 \tan\gamma}\right)^{-1} \beta y_0, \\ &\delta = \frac{\tan\gamma}{\beta}\frac{\coth\beta y_0 - 1}{\coth\beta y_0}, \quad C_1 = \frac{\beta^2 \coth\beta y_0}{\coth\beta y_0 - 1}, \quad \beta = (Bo_m - 1)^{\frac{1}{2}}. \end{aligned}\right\} \tag{8}$$

For large values of the magnetic Bond number (large Bo_m corresponds to $\gamma \ll 1$, $\delta \ll 1$), we obtain

$$y_0 = \frac{z_0}{R} = \frac{Bo_m^{\frac{1}{2}} V}{4\pi R^3 \tan\gamma}, \quad \delta = \frac{\tan\gamma}{Bo_m^{\frac{1}{2}}}.$$

In the above relationships, $y_0 = z_0/R$ determines the half-length of the droplet and δ determines the value of a. The length of the droplet z_0 is proportional to, and its thickness is inversely proportional to, the square root of Bo_m. With an increase in Bo_m the droplet shrinks in the transverse direction and spreads in the longitudinal one. The droplet thickness increases as the wetting angle increases, the other conditions being the same.

In the limit $\gamma \to 0$, the droplet degenerates into a hollow cylindrical layer adjacent to the inner side of the cylindrical clearance. Such an equilibrium position in the field of a conductor (in the absence of an electrical field) can only be occupied by a diamagnetic fluid (figure 2).

We shall consider the stability of the above-mentioned hollow cylindrical layer of the DMF with respect to constricted axisymmetric perturbations of the surface in the form

$$\zeta = \xi - a = e_0 \exp[\mathrm{i}(kz - \omega t)],$$

which are running waves with wavelength $\lambda = 1/k$, where $\zeta = \xi - a$ is the deviation of the perturbed surface from the unperturbed one, $r = a$. Viscous forces were neglected here.

It is assumed that there is a potential of the velocity of the perturbed motion of the fluid, i.e. $v = \operatorname{grad}\phi$. Then the continuity equation has the form $\nabla^2\phi = 0$.

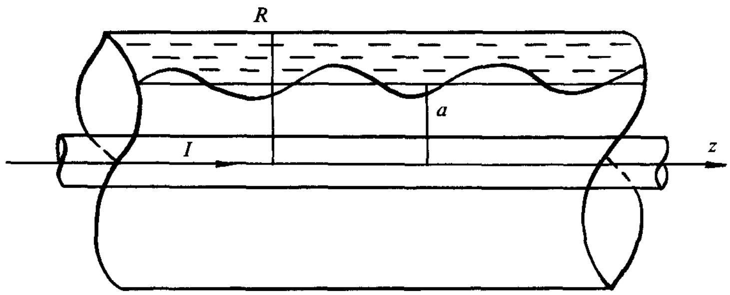

FIGURE 2. A hollow cylindrical layer of the magnetic fluid adjacent to the inner surface of the clearance in the magnetic field of a linear conductor coaxial with the clearance. One can see constricted perturbations of the surface.

Since the force $\mu_0 M\nabla H$ is of a potential nature, the Cauchy–Lagrange integral is also valid in the case of a fluid:

$$p+\rho\frac{\partial\phi}{\partial t}+\tfrac{1}{2}\rho(\nabla\phi)^2-\mu_0\int_0^H M\,\mathrm{d}H = \text{const}.$$

A similar expression has been derived for ferromagnetic fluids (Neuringer & Rosensweig 1964). Since the lines of force of the magnetic field are parallel to the disturbed surface, the equation of the jump in pressure has no magnetic term and acquires the form

$$p = p_1+\sigma(K_1+K_2).$$

The kinematic conditions at all the boundaries of the fluid

$$\frac{\partial\xi}{\partial t}+(\nabla\phi)(\nabla\xi) = \frac{\partial\phi}{\partial r} \tag{9a}$$

in the approximation of small perturbations ζ take the following form for the free surface:

$$v_r = \frac{\partial r}{\partial t}, \tag{9b}$$

and for the solid surface:

$$\left.\frac{\partial\phi}{\partial r}\right|_{r=R} = 0. \tag{9c}$$

Linearization of the system (9) with respect to small ζ results in the following set of equations:

$$\sigma[(a^{-2}-k^{-2})-Bo_{\mathrm{m}}\,a^{-2}]\,e_0-\mathrm{i}\omega\rho I_0(ka)\,C_1-\mathrm{i}\omega K_0(ka)\,C_2 = 0,$$
$$\mathrm{i}\omega e_0+kI_1(ka)\,C_1-kK_1(ka)\,C_2 = 0,$$
$$kI_1(kR)\,C_1-kK_1(kR)\,C_2 = 0,$$

where the magnetic Bond number is

$$Bo_{\mathrm{m}} = \frac{\mu_0\,|\chi|I^2}{4\pi^2\sigma a}.$$

If this set of equations has a non-zero solution, i.e. its determinant is zero, we arrive at the dispersion equation

$$\Omega^2 = s(Bo_m + s^2 - 1)\frac{I_1(kR)K_1(ka) - I_1(ka)K_1(kR)}{I_0(ka)K_1(kR) + I_1(kR)K_0(ka)},$$

where $\Omega = (\rho a^3\omega^2/\sigma)^{\frac{1}{2}}$ is the dimensionless frequency and $s = ka$ is the dimensionless wavenumber. The fraction here is positive and therefore

$$\operatorname{sgn}\Omega^2 = \operatorname{sgn}(Bo_m + s^2 - 1).$$

For $Bo_m > 1$, Ω is real and the hollow layer of the DMF is stable with respect to the constricted perturbations of the surface. For $Bo_m < 1$, we can always find a wavenumber s such that $(Bo_m - 1 + s^2) < 0$, i.e. frequencies Ω and ω are imaginary quantities, the boundary perturbation ζ monotonically increases with time and the layer is unstable with respect to the constricted perturbations of the surface whose dimensionless wavelength $\Lambda = 1/s$ exceeds $(1 - Bo_m)^{-\frac{1}{2}}$.

It follows from the critical value of the Bond number that the stability of the layer increases with an increase in the absolute value of magnetic susceptibility and the current, and drops with an increase of the surface tension and the radius of the free surface. Of interest is the fact that the layer stability depends on the radius a of the layer surface but is independent of the radius R of the clearance walls, the latter indicating that the instability under consideration is an instability of the surface.

The above-considered instability of an infinite cylindrical layer of DMF adjacent to the inner side of the cylindrical clearance in the state of weightlessness is a model problem of an important technological process, i.e. that of the fluid flow over the inner surface of the vertical cylindrical clearance in the gravitational field. In the absence of forces of electromagnetic origin such a cylindrical layer of the fluid is always unstable with respect to the above-considered constricted perturbations of the surface. The phenomenon of the stabilization of a cylindrical DMF layer in the field of a linear conductor carrying current described above allows the creation of new hydrodynamic modes in strong magnetic fields using a series of DMFs. A cylindrical DMF of radius $a = 1$ m possessing 'moderate' magnetic properties ($\chi = -10^{-5}$, $\sigma = 10^{-2}$ N/m) in the field of a current $I = 10^7$ A corresponds to the magnetic Bond number $Bo_m \sim 10^4$, which essentially exceeds its critical value and therefore the layer is stable. A current whose field retains such a layer in the stable state has the critical value $I_* = 10^5\pi^{\frac{1}{2}}$, i.e. is one–two orders of magnitude smaller than the maximum attainable currents. The attainable currents, $I = 10^7$ A, may retain in the stable state layers as thick as $a = 1$ m for a number of DMFs (e.g. of hydrogen, helium, argon, water, etc.) including very weak magnetic DMFs with $\chi = -10^{-9}\pi$.

In the limiting case of an infinitely thick hollow layer ($R = \infty$) with the condition at distant boundaries $p|_{r=R=\infty} = p_\infty$ the fluid has the pressure

$$p = \frac{\mu_0\chi I^2}{4\pi^2 r^2} + p_\infty.$$

If the radius a of the conductor is such that

$$\frac{\mu_0\chi I^2}{4\pi^2 a^2} + p_\infty < 0$$

(i.e. $p_\infty < 10^3/\pi$ Pa for $I = 10^7$ A, $\chi = -10^{-6}$, $a = 0.1$ m (Knoepfel 1970)), in the diamagnetic fluid cylindrical caverns may form around a sufficiently thin superconductor carrying a strong current (the above-described cylindrical layer). As far as we know, such a phenomenon, i.e. the drying of a conductor inside a non-conducting fluid, can take place only in diamagnetic fluids.

5. Hydrodynamic effects in uniform magnetic fields

The behaviour of DMFs in applied uniform magnetic fields is similar in many respects to that of paramagnetic fluids (PMFs).

A single PMF droplet placed in a vertical uniform magnetic field in the absence of any other body forces acquires the shape of an ellipsoid of rotation (Berkovsky *et al.* 1985; Berkovsky & Smirnov 1984, 1985). We assume that a DMF droplet in a state of weightlessness also acquires an axisymmetric shape in an applied vertical uniform field H_e which is close to the more general shape of an ellipsoid of rotation, either elongated or flattened. Then the magnetic field inside the droplet, H_i, is also uniform, H_i = const. In this case the Laplace condition for the pressure difference (2) reduces to the form

$$p_u^0 - p_b^0 - \tfrac{1}{2}\mu_0 \frac{M_b^2 - M_u^2}{1+z'^2} = \frac{\sigma}{r}\left[\frac{rz'}{(1+z'^2)^{\frac{1}{2}}}\right]';$$

where subscripts u and b denote the fluid above and under the surface of the DMF and the prime denotes differentiation with respect to r. The solution of the above equation is

$$\left.\begin{aligned} z &= \int \frac{C_1 \operatorname{sgn}(z') I_1(C_1 r)}{[\Pi^2 I_0^2(C_1 r) - C_1^2 I_1^2(C_1 r)]^{\frac{1}{2}}} \mathrm{d}r + C_3, \\ \operatorname{sgn}(z') &= -\operatorname{sgn}\{\Pi I_0(\rho)\,[I_0(\rho)]'_\rho\}, \quad \rho = C_1 r, \\ \Pi &= \frac{\mu_0(M_b^2 - M_u^2)\,L}{2\sigma}, \end{aligned}\right\} \tag{10}$$

where I_ν is the modified Bessel function; integration in (10) is carried out over any interval of r where the square-root expression in (10) is positive. It follows from (10) that the DMF droplet is elongated in the direction of the lines of force of the magnetic field similarly to the case of a droplet of a PMF. This effect should be well pronounced in experiments with droplets of true (natural) fluids which have low coefficients of surface tension σ (e.g. at high temperatures) in strong magnetic fields. Thus in the field $H = 10^7$ A/m a droplet of a typical DMF ($\chi = -10^{-5}$, $\sigma = 10^{-3}$ N/m) which has the characteristic size $L = 10^{-1}$ m, i.e. the volume $V = L^3 = 10^{-3}$ m^3, is characterized by the dimensionless parameter $\Pi = 1.256$; thus the droplet is elongated, i.e. its length-to-diameter ratio is $e = 1.28$.

Such droplet elongation (of the order of several tens of per cent and more) is observed within a certain range of parameters for a series of DMFs (hydrogen, neon, argon, water) in fields $H \sim 10^7$ A/m.

Berkovsky & Smirnov (1984) suggested a solution for the shape of a free jet of fluid in a magnetic field with the field strength vector normal to the jet. It can be shown that the cross-section of the DMF jet elongates in the direction of the field and that this effect is of the same order of magnitude as that for an elongated droplet.

The analysis of the stability of a free DMF surface in a vertical gravitation and uniform magnetic fields by the method suggested by Berkovsky *et al.* (1985) shows that, similar to the PMF surface, the DMF surface shows a step instability in magnetic fields with strength H exceeding the critical value, $H > H'_*$. Then the following relationship is valid:

$$H_*^2 = \frac{2(\mu+1)}{\mu_0 \mu \chi^2}((\rho_1 - \rho_2) g\sigma)^{\frac{1}{2}}.$$

6. Conclusions

The solutions obtained demonstrate magnetodynamic effects caused by the magnetization of fluids (helium, water, oxygen, nitrogen, etc.) which are pronounced in high magnetic fields (exceeding 1 T) in problems concerning the determination of equilibrium forms and their stability. Such effects are especially strong in non-uniform magnetic fields. Mathematically, this is explained by the fact that the dimensionless criteria proportional to those terms in (1)–(3), which are related to the hydrodynamic effects of magnetization of natural fluids in non-uniform magnetic fields (Bo_{m} for the above procedure for deriving dimensionless expressions), depend linearly on the small quantity χ, which for natural fluids is usually less than 10^{-3}. The criteria for natural fluids associated with the effects in uniform fields, e.g. We_{m}, depend on χ^2. The behaviour of fluids with negative and positive susceptibilities show some similar and different features. Both types of fluids forming jets and droplets elongate in the direction of the lines of force of the uniform magnetic field; the flat horizontal free surface of these fluids in vertical gravitational and uniform magnetic fields show step instability. At the same time, diamagnetic fluids may also take a stable equilibrium shape which cannot be obtained for other fluids, e.g. that of a hollow cylindrical layer in the non-uniform field of a linear conductor.

The fact that both diamagnetic and paramagnetic fluids show the same properties in uniform fields and different properties in non-uniform fields is mathematically explained by the linear and quadratic dependence of the main dimensionless parameters on magnetic susceptibility χ.

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